A Parametric Method Applied to Phase Recovery from a Fringe Pattern based on a Particle Swarm Optimization

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Abstract. A parametric method to carry out fringe pattern demodulation by means of a particle swarm optimization is presented. The phase is approximated by the parametric estimation of an nth-grade polynomial so that no further unwrapping is required. On the other hand, a diferent parametric function can be chosen according to the prior knowledge of the phase behavior. A particles swarm is codified with the parameters of the function that estimates the phase. A fitness function is established to evaluate the particles, which considers: (a) the closeness between the observed fringes and the recovered fringes, (b) the phase smoothness, (c) the prior knowledge of the object as its shape and size. The swarm of particles evolves until a fitness average threshold is obtained. The method can demodulate noisy fringe patterns and even a one-image closed-fringe pattern successfully.

Keywords: Phase retrieval; Fringe analysis; Optical metrology; Particle Swarm Optimization.

1 Introduction

In optical metrology, a fringe pattern (interferogram) can be represented using the following mathematical expression:

$$I(x,y) = a(x,y) + b(x,y) \times \cos\left(\omega_x x + \omega_y y + \phi(x,y) + n(x,y)\right) \tag{1}$$

where *x, y* are integer values representing indexes of the pixel location in the fringe image, *a(x,y)* is the background illumination, *b(x,y)* is the amplitude modulation and is $\phi(x,y)$ the phase term related to the physical quantity being measured. ω_x and ω_y are the angular carrier frequency in directions x and y. The term $n(x,y)$ is an additive phase noise. The purpose of any interferometric technique is to determine the phase term, which is related to the physical quantity, being

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measured. One way to calculate the phase term $\phi(x,y)$ is by using the phase-shifting technique (PST) [1–5], which needs at least three phase-shifted interferograms. The phase shift among interferograms must be known and experimentally controlled. This technique can be used when mechanical conditions are met throughout the interferometric experiment.

On the other hand, when the stability conditions mentioned are not covered, there are many techniques to estimate the phase term from a single fringe pattern, such as: the Fourier method [6,7], the Synchronous method [8] and the phase locked loop method (PLL) [9], among others. However, these techniques work well only if the analyzed interferogram has a carrier frequency, a narrow bandwidth and the signal has low noise. Moreover, these methods fail for phase calculation of a closed-fringe pattern. Additionally, the Fourier and Synchronous methods estimate the phase wrapped because of the arctangent function used in the phase calculation, so an additional unwrapping process is required. The unwrapping process is difficult when the fringe pattern includes high amplitude noise, which causes differences greater than 2π radians between adjacent pixels [10–12]. In the PLL technique, the phase is estimated by following the phase changes of the input signal by varying the phase of a computer-simulated oscillator (VCO) such that the phase error between the fringe pattern and VCO's signal vanishes.

Recently, regularization [13–15] and neural networks techniques [16,17] have been used to work with fringe patterns, which contain a narrow bandwidth and noise. The regularization technique establishes a cost function that constrains the estimated phase using two considerations: (a) fidelity between the estimated function and the observed fringe pattern and (b) smoothness of the modulated phase field. For example, in [13,18], the cost function considers a neighborhood of the pixel under analysis to fit a plane. The plane is determined by using gradient descent technique, which takes the partial derivatives of the cost function with reference local phase and carrier frequency into consideration. Its main drawback is that it can easily fall on a local minimum due to use of local gradients in the case of interferograms generated by phase fields that contain many minimal and/or maximal. In a recent work, Servin et al. [18] proposed the fringe-follower regularization phase tracker technique (FFRPT), where a scanning strategy is used to avoid the last drawback. A disadvantage of this is that a low-pass filtering and a binary threshold operation are required. These operations depend on the form of the particular fringe pattern image. Additionally, the FFRPT could be affected by noise presence due to the local consideration (taking a small neighborhood) to fit a plane to each central pixel in the image.

In the neural network technique, a multi-layer neural network (MLNN) is trained by using a set of fringe patterns and a set of phase gradients provided from calibrated objects. After the MLNN has been trained, the phase gradient is estimated in the MLNN output when the fringe patterns (interferograms) are presented in the MLNN input. The drawback of this technique is the requirement of a set of training fringe images and their related phase measurements.

In this work, we propose a technique to determine the phase $\phi(x,y)$, from a fringe pattern with a narrow bandwidth and/or noise, by parametric estimation of a global non-linear function instead of local planes in each site *(x,y)* as it was proposed in [13,18]. A particle swarm optimization (PSO) [19–21] is used to fit the best non-linear function to the phase from the full image not a small neighbourhood as in regularization techniques. The PSO technique was selected to optimize the cost function instead of gradient descent technique since non-convergence problems are presented when it is used in a non-linear function fitting. PSO strategy reduces the possibility of falling in a local optimum. When a noisy closed fringe pattern is demodulated, neither a low-pass filter nor a binarizing operator is required. On the other hand, regularization techniques need both of them.

2 PSO Applied to Phase Recovery

As described by Eberhart and Kennedy, the PSO algorithm is an adaptive algorithm based on a social-psychological metaphor; a population of individuals (referred to as particles) adapts by returning stochastically toward previously successful regions.

The fringe demodulation problem is a difficult problem to solve when the noise in the fringe pattern is high, since many solutions are possible even for a single noiseless fringe pattern. Besides, the complexity of the problem is increased when a carrier frequency does not exist (closed fringes are presented).

Given that for a closed fringe interferogram there are multiple phase functions for the same pattern, the problem is stated as an ill-posed problem in the Hadamard sense, since a unique solution cannot be obtained [22]. It is clear that image of a fringe pattern $I(x,y)$ will not change if $\phi(x,y)$ in Eq. (1) is replaced with another phase function $\bar{\phi}(x,y)$ given by

$$\bar{\phi}(x,y)=\begin{cases}-\phi(x,y)+2\pi & (x,y)\in R,\\ \phi(x,y) & (x,y)\notin R\end{cases} \tag{2}$$

where R is an arbitrary region and k is an integer. In this work, a PSO is presented to carry out the optimization process, where a parametric estimation of a non-linear function is proposed to fit the phase of a fringe pattern. Then, PSO technique fit a global non-linear function instead of a local plane to each pixel just like it is made in regularization techniques [13,18]. The fitting function is chosen depending on the prior knowledge of the demodulation problem as object shape, carrier frequency, pupil size, etc. When no prior information about the shape of $\phi(x,y)$ is known, a polynomial fitting is recommended. In this paper, authors have used a polynomial fitting to show how the method works.

The purpose in any application of PSO is to evolve a particle swarm of size P (which codifies P possible solutions to the problem) using update velocity and position of each particle, with the goal of optimizing a fitness function adequate to the problem to solve.

In phase demodulation from fringe patterns, the phase data can be approximated by the selection from one of several fitting functions, such as:

$$f_1(a,x,y)=a_0+a_1x+a_2y+a_3x^2+a_4xy+a_5y^2+\ldots+a_{\frac{(n+1)(n+2)}{2}}y^n, \tag{3}$$

$$f_2(a,x,y)=\sum_{i=1}^{N}a_i\exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{\sigma}\right] \tag{4}$$

or

$$f_3(a,x,y)=\sum_{j=0}^{n}\sum_{k=-j}^{j}a_{jk}R_j^{|k|}(p)e^{ik\theta} \tag{5}$$

among others. Parameter a is the coefficient function vector or matrix depending on the selected function, which will be estimated by the PSO (whose solution is codified inside of the particle). The function is selected on the basis of background experiment information. For example, if the phase to be calculated is smooth a loworder two-dimensional polynomial may be fitted. In this work, the fitness function U, which is used to evaluate the pth particle a^p in the swarm, is given by

$$\begin{aligned}U(a^p)=\alpha-\sum_{y=1}^{R-1}\sum_{x=1}^{C-1}&\{(I_N(x,y)\\&-\cos(\omega_x x+\omega_y y+f(a^p,x,y)))^2\\&+\lambda[(f(a^p,x,y)-f(a^p,x-1,y))^2\\&+(f(a^p,x,y)-f(a^p,x,y-1))^2]\}m(x,y),\end{aligned} \tag{6}$$

where x, y are integer values representing indexes of the pixel location in the fringe image. Superindex p is an integer index value between 1 and P, which indicates the number of chromosome in the population. $I_N(x,y)$ is the normalized version of the detected irradiance at point (x,y). The data were normalized in the range $[-1,1]$. ω_x and ω_y are the angular carrier frequencies in directions x and y. The function $f(\cdot)$ is the selected fitting function to carry out the phase approximation. $R\times C$ is the image resolution where fringe intensity values are known and λ is a smoothness weight factor (it should be clear for the reader that a higher value of parameter λ implies a smoother function to be fitted). The binary mask $m(x,y)$ is a field which defines the valid area in the fringe pattern. The parameter a can be set to the maximum value of the second term (in negative sum term) at Eq. (6) in the first chromosome population, which is given by

$$\alpha = \max_{p}\left\{\sum_{y=1}^{R-1}\sum_{x=1}^{C-1}\{(I_N(x,y) - \cos(\omega_x x + \omega_y y + f(a^p,x,y)))^2 + \lambda\left[(f(a^p,x,y) - f(a^p,x-1,y))^2 + (f(a^p,x,y) - f(a^p,x,y-1))^2\right]\}m(x,y),\right. \tag{7}$$

then parameter α is used to convert the proposal from minimal to maximal optimization since a fitness function in a PSO is considered to be a nonnegative figure of merit and profit [19].

The first term (in negative sum term) at Eq. (6) attempts to keep the local fringe model close to the observed irradiances in least-squares sense. The second term (in negative sum term) at Eq. (6) is a local discrete difference, which enforces the assumption of smoothness and continuity of the detected phase.

2.1 Particles

At the beginning of a PSO, a set of random solutions are codified in a particle swarm of size P. Each particle a is formed by the parameter function vector (possible solution) and chained string such as

$$a = \left[a_0 \,\vdots\, a_1 \,\vdots\, a_2 \,\vdots \ldots \vdots\, a_n\right] \tag{8}$$

Each dimension a_i is a random real number in a defined search range $(\min(a_i), \max(a_i))$ (the userdefined maximum and minimum of a_i). These values can be initialized using prior knowledge (e.g. in the polynomial case, components x and y are related to the interferogram tilt so if a closed fringe is presented, then these values are near 0). Every dimension is generated as

$$a_i = random(\min(a_i), \max(a_i)) \tag{9}$$

The next iterations of particles, positions and velocities are adjusted, and the function is evaluated with the new coordinates at each time-step.

2.2 Particle Velocity and Position Update

During each generation each particle is accelerated toward the particle's previous best position and the global best position. At each iteration a new velocity value for each particle is calculated based on its current velocity, the distance from its previous best position, and the distance from the global best position. The new velocity value is then used to calculate the next position of the particle in the search space. This process is then iterated a set number of times, or until a minimum error is achieved.

In the inertia version of the algorithm an inertia weight, reduced linearly each generation, is multiplied by the current velocity and the other two components are weighted randomly to produce a new velocity value for this particle, this in turn affects the next position of the particle during the next generation. Thus, the governing equations are:

$$v_{id}(t+1) = \omega \cdot v_{id} + c_1 \cdot \varphi_1 \cdot (P_{lid} - a_{id}(t)) + c_2 \cdot \varphi_2 \cdot (P_{gd} - a_{id}(t)) \tag{10}$$

$$a_{id}(t+1) = a_{id}(t) + v_{id}(t+1) \tag{11}$$

where a_i is particle i's position vector, v_i is particle i's velocity vector, c_1 and c_2 are positive constants, are called acceleration coefficients, φ_1 and φ_2 are random positive numbers between 0 and 1. Some researchers have found out that setting c_1 and c_2 equal to 2 gets the best overall performance, where as ω is called inertia weight. P_l is the local best solution found so far by the i-th particle, while P_g represents the positional coordinates of the fittest particle found so far in the entire community. Once the iterations are terminated, most of the particles are expected to converge to a small radius surrounding the global optima of the search space.

2.3 PSO Convergence

The PSO convergence mainly depends on the population size. It should be clear that if we increase the population size, more chromosomes will search the global optimum and a best solution will be found in a minor number of iterations, although the processing time can be increased [19,20]. A good rule of thumb for swarm size is to choose as large a population size as computer system limitations and time constraints allow.

To stop the PSO process, different convergence measures can be employed. In this paper, we have used a relative comparison between the fitness function value of the *gbest* particle in the swarm and value a, which is the maximum possible value to get in Eq. (6). Then, we can establish a relative evaluation of uncertainty to stop the PSO as

$$\left| \frac{\alpha - U(a^{\bullet})}{\alpha} \right| \leq \varepsilon, \tag{12}$$

where $U(a^{\bullet})$ is the fitness function value of the *gbest* particle a in the swarm in the current iteration, and ε is the relative error tolerance. Additionally, we can stop the process in a specified number of iterations, if Eq. (12) is not satisfied.

3 Experiments

The parametric method using a PSO was applied to calculate phase from three different kinds of fringe patterns: shadow moiré closed fringe pattern. We use a particles swarm size equal to 100, inertia a number in the range $[0.1, 0.9]$, velocity a number in the range $[0.0001, 0.0009]$. In each particle, the coded coefficients of a fourthgrade polynomial were included. The following polynomial was coded in each particle:

$$\begin{aligned} p_4(x,y) = {} & a_0 + a_1x + a_2y + a_3x^2 + a_4xy \\ & + a_5y^2 + a_6x^3 + a_7x^2y + a_8xy^2 \\ & + a_9y^3 + a_{10}x^4 + a_{11}x^3y + a_{12}x^2y^2 \\ & + a_{13}xy^3 + a_{14}y^4 \end{aligned} \quad (13)$$

so that 15 coefficients were configured in each particle inside swarm to be evolved.

3.1 Close Fringe Pattern

A low contrasted noisy closed fringe pattern was generated in the computer using the following expression:

$$I(x,y) = 127 + 63\cos(P_4(x,y) + \eta(x,y)), \quad (14)$$

where

$$\begin{aligned} p_4(x,y) = {} & -0.7316x - 0.2801y + 0.0065x^2 \\ & + 0.00036xy - 0.0372y^2 \\ & + 0.00212x^3 + 0.000272x^2y \\ & + 0.001xy^2 - 0.002y^3 \\ & + 0.000012x^4 + 0.00015x^3y \\ & + 0.00023x^2y^2 + 0.00011xy^3 \\ & + 0.000086y^4 \end{aligned} \quad (15)$$

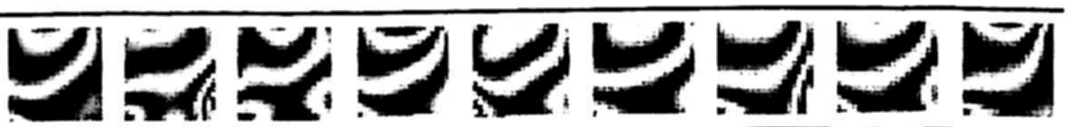
and $\eta(x,y)$ is the uniform additive noise in the range $[-2radians, 2radians]$. Additionally, the fringe pattern was generated with a low resolution of 60×60. In this case, we use a parameter search range of $[-1,1]$. The swarm of particles was evolved until the number of iterations and relative error tolerance ε was 0.05 in Eq. (12). This condition was achieved in a time of 60s on a AMD Phemon X4-2.5 GHz computer. The fringe pattern and the contour phase field of the computer generated interferogram are shown in Fig. 1. The PSO technique was used to recover the phase from the fringe pattern. The fringe pattern and the phase estimated by PSO is shown in Fig. 1. The normalized RMS error was 0.12 radians and the peak-to-valley error was 0.94 radians. The test as shown in Table 1, and shows best particle for the testers shows in Table 2.

Table 1. Table of parameters of inertial and velocity

	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.0001	2.870	3.432	3.612	3.505	3.839	3.277	2.916	2.777	2.395
0.0002	3.007	3.044	3.210	3.083	2.725	2.680	1.688	1.801	2.366
0.0003	1.665	1.875	2.565	2.559	1.576	1.708	**1.151**	1.945	2.469
0.0004	2.17	**1.738**	2.777	1.912	**1.290**	2.171	1.806	**0.567**	1.946
0.0005	1.883	1.860	2.838	1.686	1.701	2.063	1.969	0.791	1.792
0.0006	2.106	2.134	2.900	**1.086**	2.318	1.705	1.645	1.399	2.343
0.0007	1.928	1.993	**0.853**	1.168	2.019	2.270	1.772	1.428	1.828
0.0008	**0.893**	1.938	1.350	1.531	2.019	2.632	1.373	1.373	2.260
0.0009	1.536	1.911	1.436	1.773	2.407	**0.313**	1.902	0.779	**1.523**

Table 2. Shows of the best particles

Inertia	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Velocity	0.0008	0.0004	0.0007	0.0006	0.0004	0.0009	0.0003	0.0004	0.0009



a) b)

Fig. 1. (a) Fringe pattern and (b) phase field obtained by using PSO technique

4 Conclusions

A PSO was applied to recover the modulating phase from closed and noisy fringe patterns. A fitness function, which considers the prior knowledge of the object being tested, is established to approximate the phase data. In this work a fourthgrade polynomial was chosen to fit the phase.

A swarm of particles was generated to carry out the optimization process. Each particle was formed by a codified string of polynomial coefficients. Then, the swarm of particles was evolved using velocity, position and inertial

The PSO technique works successfully where other techniques fail (Synchronous and Fourier methods). This is the case when a noisy, wide bandwidth and/or closed fringe pattern is demodulated. Regularization techniques can be used in these cases but PSO technique has the advantage that the cost function does not depend upon the existence of derivatives and restrictive requirements of continuity (gradient descent methods). Since the PSO works with a swarm of possible solutions instead of a single solution, it avoids falling in a local optimum. Additionally, no filters and no binarizing operators are required, in contrast with the fringe-follower regularized phase tracker technique.

The PSO has the advantage that if the user knows prior knowledge of the object shape, then a better suited fitting parametric function can be used instead of a general polynomial function. Additionally, due to the fact that the PSO technique gets the parameters of the fitting function, it can be used to interpolate sub-pixel values and to increase the original phase resolution or interpolate where fringes do not exist or are not valid. A drawback is the selection of the optimal initial PSO parameters (such as swarm size, inertial, velocity) that can increase the convergence speed.

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